HELIOSEISMIC DETERMINATION OF OPACITY CORRECTIONS

S.C. Tripathy[1], Sarbani Basu[2] and J. Christensen-Dalsgaard[2,3]

[1]Udaipur Solar Observatory, Physical Research Laboratory, PO Box No. 198, Udaipur 313 001, India
[2]Teoretisk Astrofysik Center, Danmarks Grundforskningsfond, DK-8000 Aarhus C, Denmark
[3]Institut for Fysik og Astronomi, Aarhus Universitet, DK-8000, Aarhus, Denmark

1. INTRODUCTION

Accurate measurements of oscillation frequencies of the Sun are providing detailed information about the structure of the Sun. These frequencies can be used to test the basic input physics, like opacities, equation of state and nuclear energy generation rates which are required to construct theoretical solar models. Here we concentrate on the effects of opacity. Throughout we consider models calibrated to solar radius and luminosity, by adjusting the initial composition and the mixing-length parameter.

2. METHOD

We consider opacity modifications given by

$$\log\kappa = \log\kappa_0 + f(T, T_0) , \qquad (1)$$

where κ_0 is the unmodified opacity, obtained as a function of T, density ρ and composition and the function $f(T, T_0)$ has the form

$$f(T, T_0) = A \exp\left[-\left(\frac{\log T - \log T_0}{\Delta}\right)^2\right] , \qquad (2)$$

where log is logarithm to base 10. The constants A and Δ set the magnitude and width of the opacity modification. By varying T_0, we can investigate the effects of changes in different parts of the models.

Assuming a linear relation between small modifications in the opacity and the response of the model (see Tripathy & Christensen-Dalsgaard 1995), the effect of an opacity change $\delta\log\kappa(T)$ for any quantity F can be approximated by the relation

$$\frac{\delta F}{F} = \int K_F(T)\delta\log\kappa(T) \mathrm{d}\log T . \qquad (3)$$

The kernel $K_F(T)$ may be estimated from the change δF corresponding to the opacity modification given in equation (1) for sufficiently small A and Δ.

For this work we consider $F = c(r)^2$, the squared sound speed; thus $\delta c^2/c^2$ is the relative squared sound-speed difference between the Sun or a test model and the reference model. This quantity can be obtained by a straightforward inversion of solar oscillation frequencies (e.g., Basu *et al.*, 1996). It can be related to an intrinsic opacity difference $\delta\log\kappa$ through equation (3), with a kernel $K_c(r, T)$ which may be estimated by applying modifications of the form given in equation (2) for a range of T_0. If the sound-speed difference between the Sun and the model is assumed to arise solely from opacity errors, these may then be determined by inversion of the relation between $\delta\log\kappa$ and $\delta c^2/c^2$.

Here we expand the opacity correction in terms of the basis functions given by equation (2), as

$$\delta\log\kappa(T) = \sum_{i=1}^{N} b_i f(T, T_i) . \qquad (4)$$

The resulting change in the squared sound speed can be written as

$$\left(\frac{\delta c^2}{c^2}\right)(r) = \sum_{i=1}^{N} b_i \psi_i(r) , \qquad (5)$$

where $\psi_i(r)$ is the change $(\delta c^2/c^2)(r)$ resulting from applying the opacity change $\delta\log\kappa = f(T, T_i)$. It was determined by carrying out a solar evolution calculation with the modified opacity. Having thus obtained $\psi_i(r)$ for a suitable set of T_i, equation (5) may be fitted to the actual $\delta c^2/c^2$ by means of a regularized least-squares fit to determine the coefficients b_i, and hence the opacity correction $\delta\log\kappa$. We have calculated $\psi_i(r)$ at 57 values of T_i, with $\log T_i = 6.2, \ldots, 7.19$, $\Delta = 0.02$ and $A = 0.02$.

The procedure was tested by applying it to differences between two models: the reference model was computed with the Cox & Tabor (1976) opacities, while the test model used the Los Alamos Opacity Library (Huebner *et al.* 1977). Frequency differences between the two models, for the mode set used in the analysis of solar data, were inverted to infer the sound-speed differences, and the result was fitted to obtain the corresponding opacity change. The resulting inferred $\delta\log\kappa$ was in good agreement with the actual intrinsic opacity difference between the two tables (at fixed T, density ρ and composition). Thus we conclude that the method allows a determination of the opacity correction required to match a given sound-speed difference.

3. RESULTS FOR SOLAR DATA

We have applied the procedure to sound-speed differences between the Sun and a solar model. The data, model and inversion results are described by Basu *et al.*, (1996). The data are a combination of low-degree BiSON data and LOWL data of degrees between 3 and 99. The reference model (Model S of Christensen-Dalsgaard *et al.* 1996) used OPAL opacities (e.g. Iglesias *et al.* 1992), and included settling

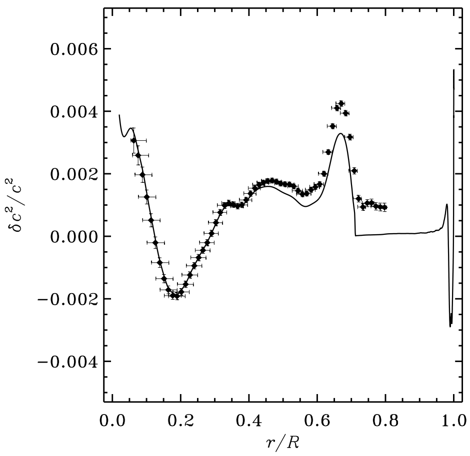

Figure 1: Comparison between the inferred difference in squared sound speed between the Sun and the model (Model S of Christensen-Dalsgaard *et al.* 1996), shown with symbols, and the model change resulting from an evolution calculation where the opacity change in Fig. 2 was added to the opacity tables. The differences are shown in the sense (Sun) – (reference model).

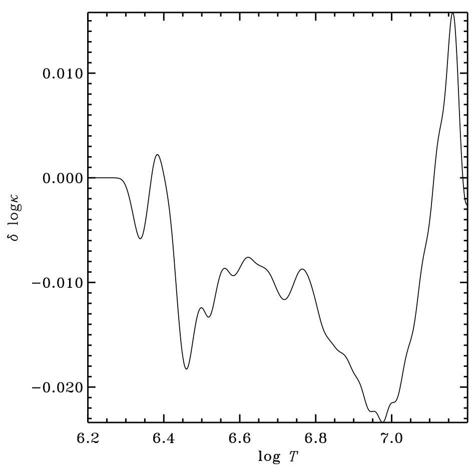

Figure 2: Opacity difference fitted from inferred sound-speed difference between the Sun and Model S of Christensen-Dalsgaard *et al.* (1996).

of helium and heavy elements. The inversion was carried out by means of the Subtractive Optimally Localized Averaging technique. The inferred differences in squared sound speed are shown in Figure 1.

The opacity modification resulting from a fit to the sound-speed difference is shown in Fig. 2. This indicates that a sharp feature in $\delta \log \kappa$ is required just beneath the convection zone, with a second peak in the core. To test that this opacity modification does in fact reproduce the inferred sound-speed difference, we have subsequently recomputed the solar evolution model, adding $\delta \log \kappa(T)$ to the opacity as interpolated from the tables. In Fig. 1, the solid line shows the resulting difference in squared sound speed, relative to the reference model. It is evident that this is in fact quite close to the sound-speed differences inferred from the inversion.

4. DISCUSSION

We have attempted to infer the corrections to the opacity required to match the sound speed in a state-of-the-art solar model to that inferred from helioseismic inversion. We find that the sound-speed difference can to a large extent be reproduced by a change in the opacity in the radiative interior. The required change, of order of a few percent, is probably within the general level of uncertainty of current opacity calculations. Since the energy transport in the solar convection zone is independent of opacity, the sound-speed difference does not give us any information about the opacity in that region.

It must be noted that the sound-speed difference between two models is essentially insensitive to a change in the opacity by a constant factor within the radiative interior. The change in opacity is largely compensated by a change in the composition, the sound-speed profile remaining approximately the same. Thus in the fit in Fig. 2, $\delta \log \kappa$ is determined only to within a constant. The freedom in the level of the relative opacity change could, for example, be utilized to choose a solution for which the change in the initial hydrogen abundance is small.

We also note that rather similar effects on the sound speed can be obtained through suitable weak mixing of the solar interior or at the base of the convection zone (Basu & Antia, 1994). Therefore, it cannot be claimed that the opacity errors are the only, or even the dominant, actual error in current solar-model calculations. A more detailed investigation of the individual contributions to the opacity, and their likely uncertainty, is required to test whether the specific shape of $\delta \log \kappa$ is realistic.

ACKNOWLEDGEMENTS

This work was supported by the Danish National Research Foundation through its establishment of the Theoretical Astrophysics Center. SCT acknowledges financial support from the Department of Science and Technology, Government of India.

REFERENCES

1. Basu, S., Antia, H. M. 1994, MNRAS 269, 1137
2. Basu, S., Chaplin, W., Christensen-Dalsgaard, J., *et al.*, *these proceedings*
3. Christensen-Dalsgaard, J., Däppen, W., Ajukov, S. V., *et al.*, 1996. *Science*, **272**, 1286
4. Cox, A. N., Tabor, J. E., 1976. *Astrophys. J. Suppl.*, **31**, 271
5. Huebner, W. F., Merts, A. L., Magee, N. H., Argo, M. F., 1977. *Astrophysical Opacity Library*, Los Alamos Scientific Laboratory Report LA-6760-M.
6. Iglesias, C. A., Rogers, F. J. & Wilson, B. G., 1990. *Astrophys. J.*, **360**, 221
7. Tripathy, S., Christensen-Dalsgaard, J. 1995, in GONG'94: Helio- and Astero-Seismology, eds. R.K. Ulrich, E. J. Rhodes, Jr. and W. Däppen, ASP Conf. Ser. **76**, 132